



Exotic Options

Chapter 24

Types of Exotics

- Package
- Nonstandard American options
- Forward start options
- Compound options
- Chooser options
- Barrier options
- Binary options
- Lookback options
- Shout options
- Asian options
- Options to exchange one asset for another
- Options involving several assets
- Volatility and Variance swaps

Packages (page 555)

- Portfolios of standard options
- Examples from Chapter 10: bull spreads, bear spreads, straddles, etc
- Often structured to have zero cost
- One popular package is a range forward contract

Non-Standard American Options (page 556)

- Exercisable only on specific dates (Bermudans)
- Early exercise allowed during only part of life (initial “lock out” period)
- Strike price changes over the life (warrants, convertibles)

Forward Start Options (page 556)

- Option starts at a future time, T_1
- Implicit in employee stock option plans
- Often structured so that strike price equals asset price at time T_1

Compound Option (page 557)

- Option to buy or sell an option
 - Call on call
 - Put on call
 - Call on put
 - Put on put
- Can be valued analytically
- Price is quite low compared with a regular option

Chooser Option “As You Like It” (page 558)

- Option starts at time 0, matures at T_2
- At T_1 ($0 < T_1 < T_2$) buyer chooses whether it is a put or call
- This is a package!

Chooser Option as a Package

At time T_1 the value is $\max(c, p)$

From put-call parity

$$p = c + e^{-r(T_2 - T_1)} K - S_1 e^{-q(T_2 - T_1)}$$

The value at time T_1 is therefore

$$c + e^{-q(T_2 - T_1)} \max(0, K e^{(r - q)(T_2 - T_1)} - S_1)$$

This is a call maturing at time T_2 plus a put maturing at time T_1

Barrier Options (page 558)

- Option comes into existence only if stock price hits barrier before option maturity
 - 'In' options
- Option dies if stock price hits barrier before option maturity
 - 'Out' options

Barrier Options (continued)

- Stock price must hit barrier from below
 - 'Up' options
- Stock price must hit barrier from above
 - 'Down' options
- Option may be a put or a call
- Eight possible combinations

Parity Relations

$$C = C_{ui} + C_{uo}$$

$$C = C_{di} + C_{do}$$

$$p = p_{ui} + p_{uo}$$

$$p = p_{di} + p_{do}$$

Binary Options (page 561)

- Cash-or-nothing: pays Q if $S_T > K$, otherwise pays nothing.
 - Value = $e^{-rT} Q N(d_2)$
- Asset-or-nothing: pays S_T if $S_T > K$, otherwise pays nothing.
 - Value = $S_0 e^{-qT} N(d_1)$

Decomposition of a Call Option

Long Asset-or-Nothing option

Short Cash-or-Nothing option where
payoff is K

$$\text{Value} = S_0 e^{-qT} N(d_1) - e^{-rT} K N(d_2)$$

Lookback Options (page 561-63)

- Floating lookback call pays $S_T - S_{\min}$ at time T (Allows buyer to buy stock at lowest observed price in some interval of time)
- Floating lookback put pays $S_{\max} - S_T$ at time T
(Allows buyer to sell stock at highest observed price in some interval of time)
- Fixed lookback call pays $\max(S_{\max} - K, 0)$
- Fixed lookback put pays $\max(K - S_{\min}, 0)$
- Analytic valuation for all types

Shout Options (page 563-64)

- Buyer can 'shout' once during option life
- Final payoff is either
 - Usual option payoff, $\max(S_T - K, 0)$, or
 - Intrinsic value at time of shout, $S_\tau - K$
- Payoff: $\max(S_T - S_\tau, 0) + S_\tau - K$
- Similar to lookback option but cheaper
- How can a binomial tree be used to value a shout option?

Asian Options (page 564)

- Payoff related to average stock price
- Average Price options pay:
 - Call: $\max(S_{\text{ave}} - K, 0)$
 - Put: $\max(K - S_{\text{ave}}, 0)$
- Average Strike options pay:
 - Call: $\max(S_T - S_{\text{ave}}, 0)$
 - Put: $\max(S_{\text{ave}} - S_T, 0)$

Asian Options

- No exact analytic valuation
- Can be approximately valued by assuming that the average stock price is lognormally distributed

Exchange Options (page 566-67)

- Option to exchange one asset for another
- For example, an option to exchange one unit of U for one unit of V
- Payoff is $\max(V_T - U_T, 0)$

Basket Options (page 567)

- A basket option is an option to buy or sell a portfolio of assets
- This can be valued by calculating the first two moments of the value of the basket and then assuming it is lognormal

Volatility and Variance Swaps

- Agreement to exchange the realized volatility between time 0 and time T for a prespecified fixed volatility with both being multiplied by a prespecified principal
- Variance swap is agreement to exchange the realized variance rate between time 0 and time T for a prespecified fixed variance rate with both being multiplied by a prespecified principal
- Daily return is assumed to be zero in calculating the volatility or variance rate

Variance Swaps (page 568-69)

- The (risk-neutral) expected variance rate between times 0 and T can be calculated from the prices of European call and put options with different strikes and maturity T
- Variance swaps can therefore be valued analytically if enough options trade
- For a volatility swap it is necessary to use the approximate relation

$$\hat{E}(\bar{\sigma}) \approx \sqrt{\frac{\hat{E}(\bar{V})}{E(V)}}$$

VIX Index (page 570)

- The expected value of the variance of the S&P 500 over 30 days is calculated from the CBOE market prices of European put and call options on the S&P 500
- This is then multiplied by $365/30$ and the VIX index is set equal to the square root of the result

How Difficult is it to Hedge Exotic Options?

- In some cases exotic options are easier to hedge than the corresponding vanilla options (e.g., Asian options)
- In other cases they are more difficult to hedge (e.g., barrier options)

Static Options Replication

(Section 24.14, page 570)

- This involves approximately replicating an exotic option with a portfolio of vanilla options
- Underlying principle: if we match the value of an exotic option on some boundary, we have matched it at all interior points of the boundary
- Static options replication can be contrasted with dynamic options replication where we have to trade continuously to match the option

Example

- A 9-month up-and-out call option on a non-dividend paying stock where $S_0 = 50$, $K = 50$, the barrier is 60, $r = 10\%$, and $\sigma = 30\%$
- Any boundary can be chosen but the natural one is

$$c(S, 0.75) = \text{MAX}(S - 50, 0) \text{ when } S < 60$$

$$c(60, t) = 0 \text{ when } 0 \leq t \leq 0.75$$

Example (continued)

We might try to match the following points on the boundary

$$c(S, 0.75) = \text{MAX}(S - 50, 0) \text{ for } S < 60$$

$$c(60, 0.50) = 0$$

$$c(60, 0.25) = 0$$

$$c(60, 0.00) = 0$$

Example continued

(See Table 24.1, page 572)

We can do this as follows:

+1.00 call with maturity 0.75 & strike 50

-2.66 call with maturity 0.75 & strike 60

+0.97 call with maturity 0.50 & strike 60

+0.28 call with maturity 0.25 & strike 60

Example (continued)

- This portfolio is worth 0.73 at time zero compared with 0.31 for the up-and out option
- As we use more options the value of the replicating portfolio converges to the value of the exotic option
- For example, with 18 points matched on the horizontal boundary the value of the replicating portfolio reduces to 0.38; with 100 points being matched it reduces to 0.32

Using Static Options Replication

- To hedge an exotic option we short the portfolio that replicates the boundary conditions
- The portfolio must be unwound when any part of the boundary is reached